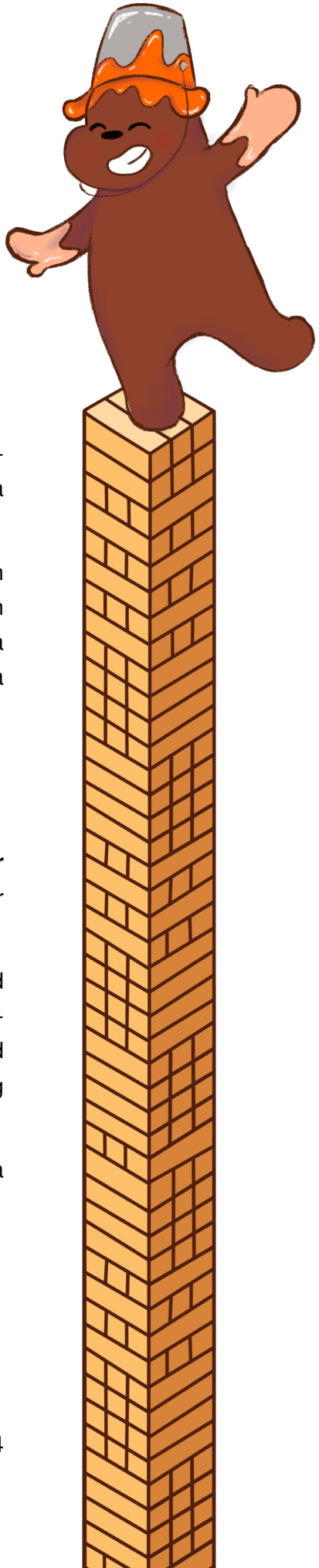
Problem 5: Big Ben's Jenga Bricks
4+1=5 Points

Problem ID: `jenga`
Rank: `2+3`



Introduction

Up for a challenge? Try solving the harder version of this problem: `benga`!

Big Ben is learning how to play Jenga! He is learning from the best players around the world: Bessie the Cow, P/NPenguin, and Ana, the Jenga National Champion from Mañusgo!

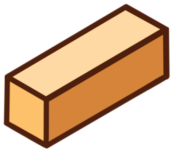
However, Big Ben wants to master the game with every initial configuration of Jenga, in other words, any possible horizontal rotation of the bricks in the game. Now he wonders how many unique Jenga towers, that use a certain amount of bricks, he needs to master before calling himself a Jenga champion.

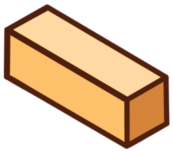
Problem Statement

Count the number of unique Jenga towers that can be built using N **or fewer** bricks of size $1 \times 1 \times 3$. A Jenga tower is a fully packed rectangular prism with a 3×3 base and a height of at least 1.

Bricks are indistinguishable from one another. Bricks can only be rotated 90° into horizontal orientations as shown below. Trivial rotations (for example, rotating a brick along its major axis) of individual Jenga bricks should not be considered unique. However, 90° rotations of the entire tower along the vertical axis should be considered unique.

To celebrate CALICO's 2-year-old birthday and the fact that bricks have a length of 3, give your answer modulo $2^{3^2}3^{2^3} = 3359232$.





Input Format

The first line of the input contains a single integer $\mathbf{T}$ denoting the number of test cases that follow.

Each test case is described in a single line containing an integer $\mathbf{N}$ denoting the number of bricks.

Output Format

For each test case, output a single line containing an integer denoting the number of unique Jenga towers modulo $2^{3^2}3^{2^3} = 3359232$.

Constraints

$1 \leq \mathbf{T} \leq 10$

Main Test Set

$1 \leq \mathbf{N} \leq 10^3$

Bonus Test Set

$1 \leq \mathbf{N} \leq 10^{18}$

Sample Test Cases

Main Sample Input Download

```
6
2
3
6
11
16
705
```

Main Sample Output Download

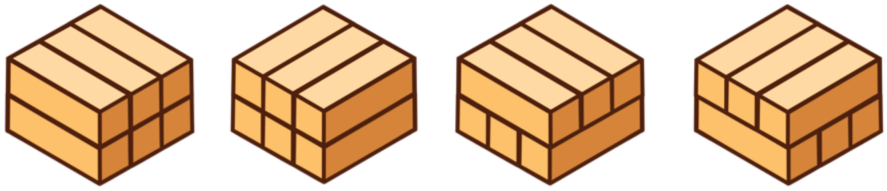
```
0
2
6
14
62
1629182
```

Main Sample Explanations

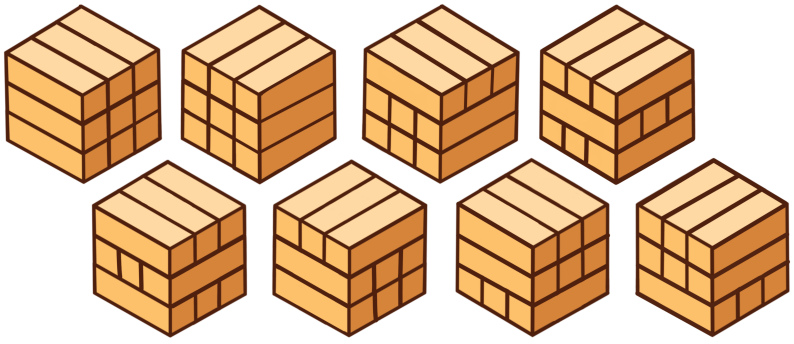
For test case #1, 2 bricks isn't enough to fill any layers, so our answer is 0. (A valid Jenga tower must have a height of at least 1.)

For test case #2, 3 bricks lets us build only towers of height 1. The only 2 ways of building towers of height 1 are shown below.

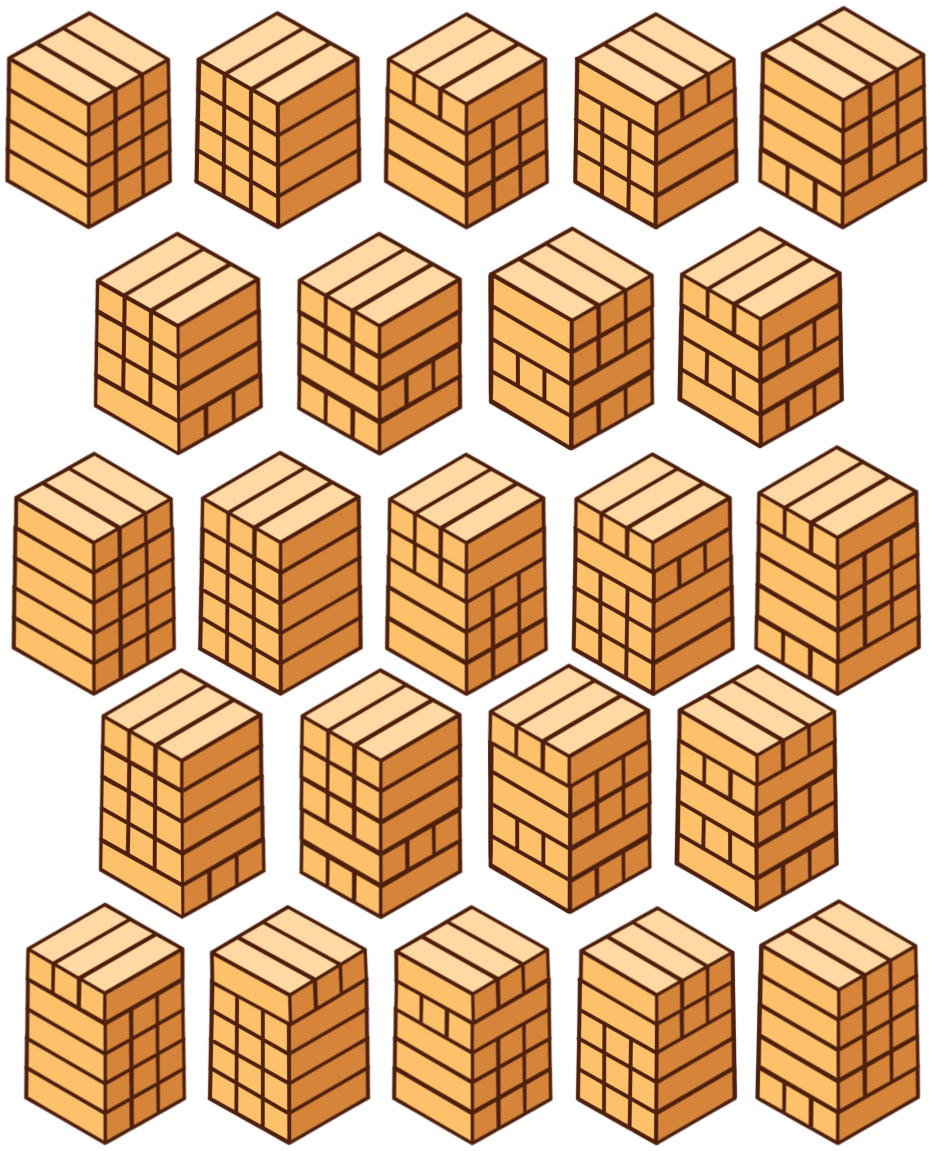
For test case #3, we can build towers of height 1 or 2. The 2 ways for height 1 are above and the 4 ways for height 2 are below, making a total of 6.



For test case #4, we can build towers of height up to 3. The 8 different towers of height 3 are shown below. This makes 14 in total.



For test case #5, there are 62 ways to build towers with height up to 5. Some (but not all) of these ways are shown below.



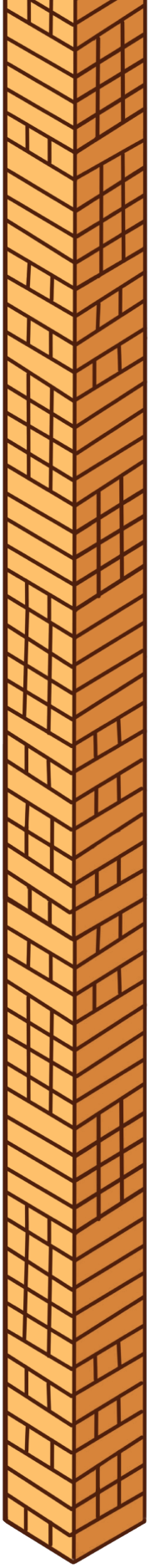
For test case #6, one of the towers that can be built using 705 bricks is shown to the right of this document.

Bonus Sample In Download

```
2
3359232
333333333333333333
```

Bonus Sample Out Download

```
2086398
1590782
```

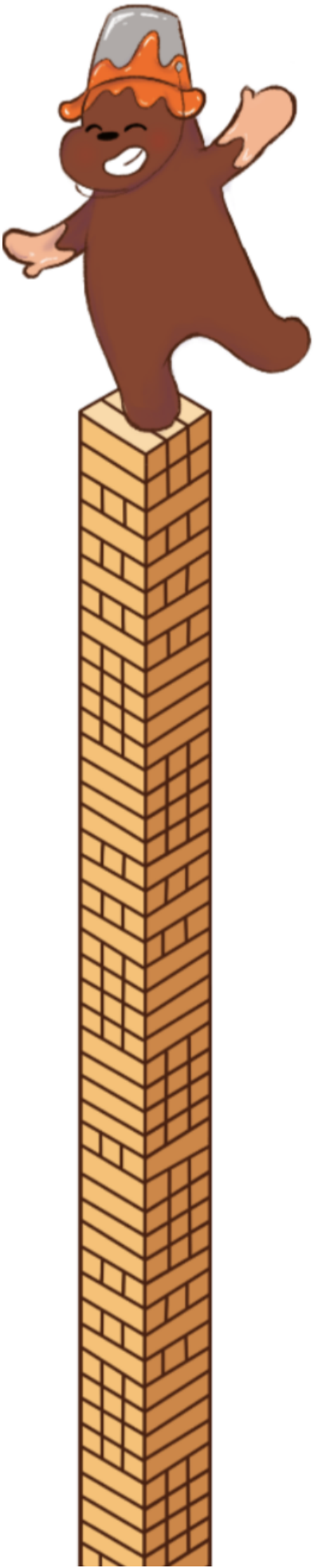


第 5 题：大本熊的叠叠高塔楼

4+1=5 分

问题标识符：jenga

难度等级：2+3



问题背景

想再来点挑战吗？试试这个问题的更难版本：benga！

大本熊正在学着玩叠叠高！他正努力从世界各地最好的玩家那里学习技巧：贝丝牛、P/NPenguin 和来自 Mañusgo 的叠叠高全国冠军 Don Matías！

现在，他想掌握叠叠高的每种初始放置方式；换句话说，就是初始时积木任何水平旋转后的放置方式。大本熊想知道他需要掌握多少个使用特定数量积木建造的独特的叠叠高塔楼之后才能成为叠叠高冠军！

问题描述

请计算使用**不超过** N 块 1 × 1 × 3 大小的积木可以建造多少个独特的叠叠高塔楼。一个叠叠高塔楼的定义为一个底面为 3 × 3 且高度至少为 1 的实心长方体。

为了庆祝 CALICO 的 2 岁生日，并且积木的长度为 3，请将答案进行取模（mod）$2^{3^2}3^{2^3} = 3359232$。

每块积木都是相同大小的。如下所示，积木只能沿水平方向旋转。你可以忽略塔楼的对称性；也就是说，即使该塔楼在垂直轴旋转或垂直平面翻转后与其他塔楼相同，这也被视为不同的设计。然而，每块积木都具有旋转对称性的，因此沿其主轴旋转 90、180 或 270 度都被视为相同的放置方式。

输入格式

输入的第一行为一个整数 T，表示后面测试用例的数量。

每个测试用例为一行，包含一个整数 N，表示积木的数量。

输出格式

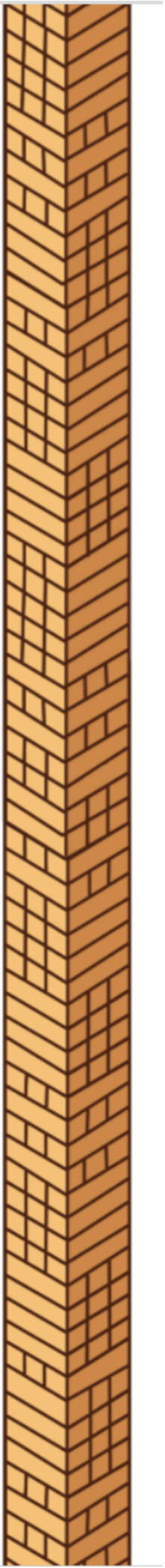
对于每个测试用例，请输出一个整数，表示独特的叠叠高塔楼数量在取模（mod）$2^{3^2}3^{2^3} = 3359232$ 后的结果。

数据范围

$1 \le T \le 10$

主测试集
$1 \le N \le 10^3$

附加测试集
$1 \le N \le 10^{18}$



测试样例

主样例输入 下载

```
6
2
3
6
11
16
705
```

主样例输出 下载

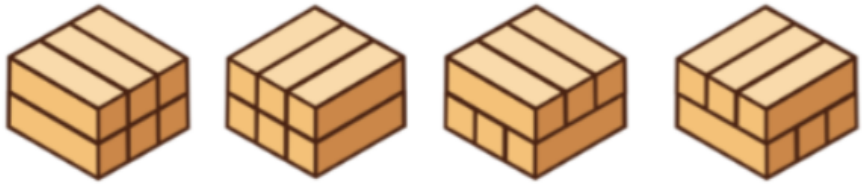
```
0
2
6
14
62
1629182
```

主样例解释

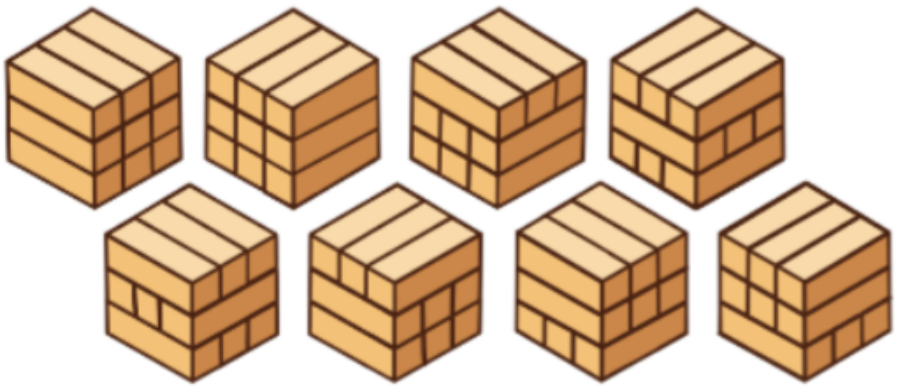
测试用例 #1：2 块积木不够建造任何一层，所以答案是 0。

测试用例 #2：3 块积木只能建造高度为 1 的塔楼。2 种方式如下所示。

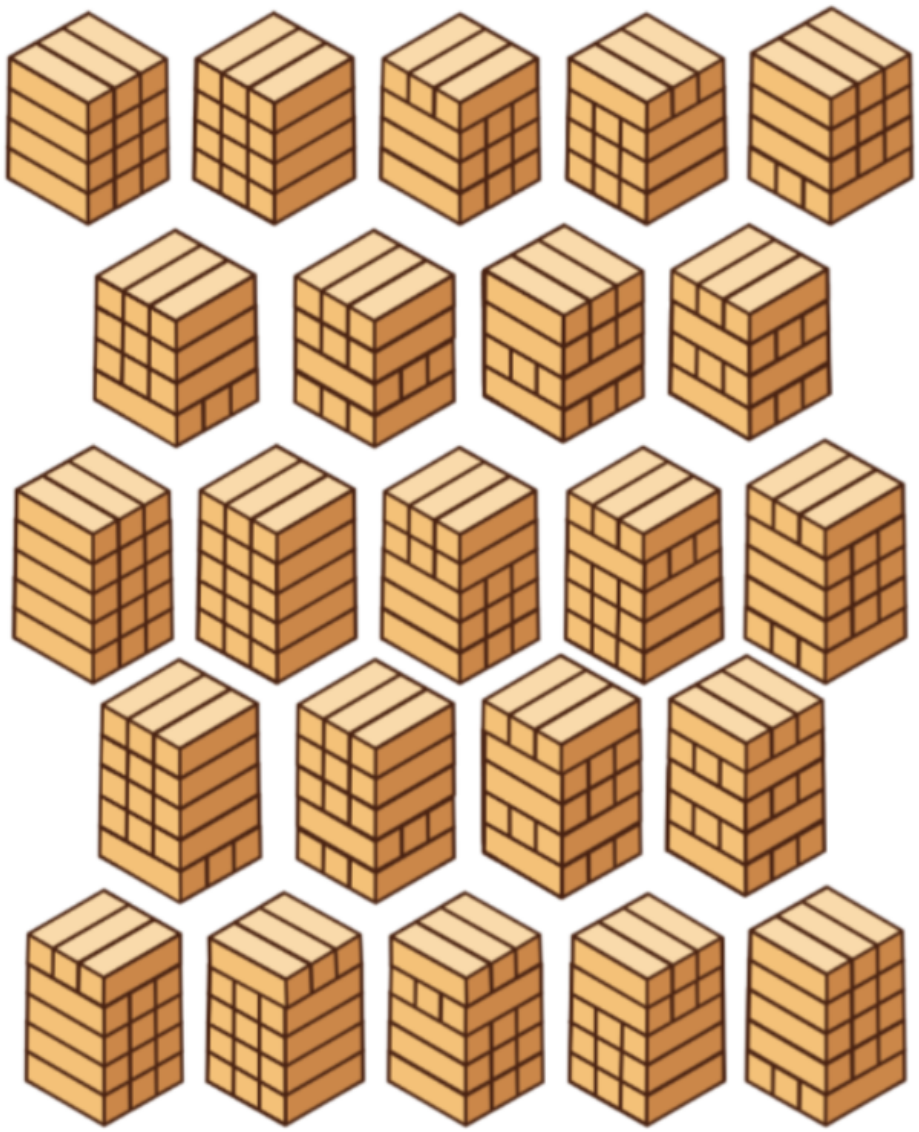
测试用例 #3：可以建造高度为 1 或 2 的塔楼。建造高度为 1 的塔楼的 2 种方式如上所示，建造高度为 2 的塔楼的 4 种方式如下所示，一共有 6 种方式。



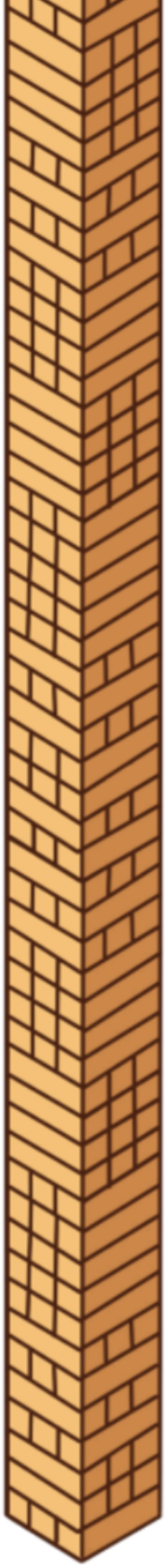
测试用例 #4：可以建造高度不超过 3 层的塔楼。建造高度为 3 的塔楼的 8 种方式如下所示。一共是 14 种方式。



测试用例 #5：有 62 种方式建造高度不超过 5 层的塔楼。下图展示了部分搭建方式。



测试用例 #6：使用 705 块积木可搭建的一个塔楼如本文件右侧所示。



附加样例输入 下载

```
2
3359232
333333333333333333
```

附加样例输出 下载

```
2086398
1590782
```