Problem 5: Big Ben's Jenga Bricks 4+1=5 Points

Problem ID: jenga Rank: 2+3

Introduction

Up for a challenge? Try solving the harder version of this problem: benga!

Big Ben is learning how to play <u>Jenga</u>! He is learning from the best players around the world: Bessie the Cow, P/NPenguin, and Ana, the Jenga National Champion from Mañusgo!

However, Big Ben wants to master the game with every initial configuration of Jenga, in other words, any possible horizontal rotation of the bricks in the game. Now he wonders how many unique Jenga towers, that use a certain amount of bricks, he needs to master before calling himself a Jenga champion.

Problem Statement

Count the number of unique Jenga towers that can be built using N or fewer bricks of size $1 \times 1 \times 3$. A Jenga tower is a fully packed rectangular prism with a 3×3 base and a height of at least 1.

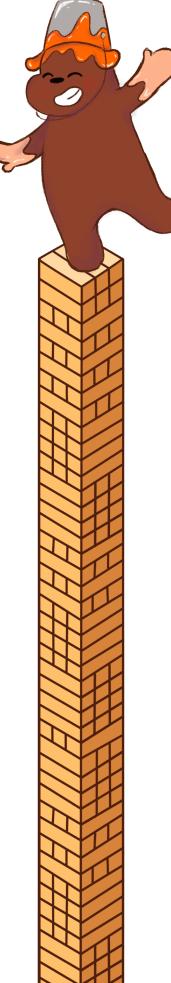
Bricks are indistinguishable from one another. Bricks can only be rotated 90° into horizontal orientations as shown below. Trivial rotations (for example, rotating a brick along its major axis) of individual Jenga bricks should not be considered unique. However, 90° rotations of the entire tower along the vertical axis should be considered unique.

To celebrate CALICO's 2-year-old birthday and the fact that bricks have a length of 3, give your answer modulo $2^{3^2}3^{2^3} = 3359232$.





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Input Format

The first line of the input contains a single integer ${\bf T}$ denoting the number of test cases that follow.

Each test case is described in a single line containing an integer ${\bf N}$ denoting the number of bricks.

Output Format

For each test case, output a single line containing an integer denoting the number of unique Jenga towers modulo $2^{3^2}3^{2^3} = 3359232$.

Constraints

 $1 \leq \mathbf{T} \leq 10$

Main Test Set

 $1 \leq \mathbf{N} \leq 10^3$

Bonus Test Set

 $1 \leq \mathbf{N} \leq 10^{18}$

Sample Test Cases

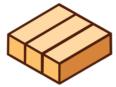
Main Sample Input	Download	Main Sample Output	<u>Download</u>
6			
2		0	
3		2	
6		6	
11		14	
16		62	
705		1629182	

Main Sample Explanations

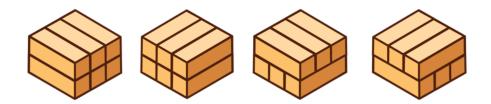
For test case #1, 2 bricks isn't enough to fill any layers, so our answer is 0. (A valid Jenga tower must have a height of at least 1.)

For test case #2, 3 bricks lets us build only towers of height 1. The only 2 ways of building towers of height 1 are shown below.

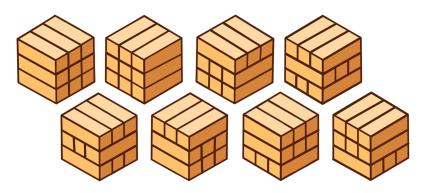




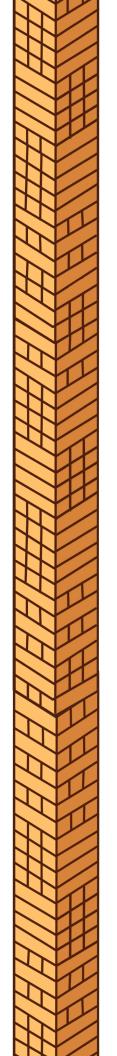
For test case #3, we can build towers of height 1 or 2. The 2 ways for height 1 are above and the 4 ways for height 2 are below, making a total of 6.



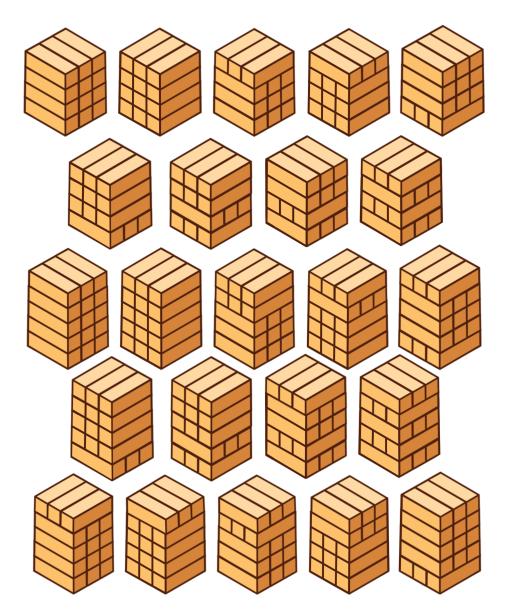
For test case #4, we can build towers of height up to 3. The 8 different towers of height 3 are shown below. This makes 14 in total.



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For test case #5, there are 62 ways to build towers with height up to 5. Some (but not all) of these ways are shown below.



For test case #6, one of the towers that can be built using 705 bricks is shown to the right of this document.

Bonus Sample In	Download	Bonus Sample Out	Download
2			
3359232		2086398	
33333333333333333333		1590782	