Problem 9: GoatC-Tree
14 Points

Problem ID: `goat`

Rank: 4

Introduction

G.O.A.T. stands for Greatest common divisor Of All Time, which is a function on a rooted tree. Your friend Bob is offering to pay you like three quintillion dollars if you can help them answer a bunch of GOAT queries for their math homework or whatever

Problem Statement

You are given N vertices (numbered 1 to N) forming a tree rooted at vertex 1. The vertices are labeled with positive integers $A_1, A_2, \ldots, A_N$, where A_i is the label of vertex i. Find the result of each of Q GOAT queries $G_1, G_2, \ldots, G_Q$, where the result of query G_i is GOAT(G_i): the sum of $\gcd(A_u, A_v)$ for all unordered pairs of vertices (u, v) modulo 998244353, such that u and v are present in vertex G_i's subtree and $u \neq v$.

Formally:

$$\text{GOAT}(G_i) = \sum_{\forall u, v \in subtree(G_i), u \neq v} \gcd(A_u, A_v) \pmod{998244353}$$

Input Format

The first line of the input contains an integer T denoting the number of test cases that follow. For each test case:

- The first line contains two space-separated integers N Q, where:
  - N denotes the size of the tree.
  - Q denotes the number of GOAT queries.
- The second line contains N space-separated integers P_1 P_2 … P_N, denoting that each vertex i's parent in the tree is vertex P_i
  - P_1 will always be zero to represent vertex 1 being the root of the tree.
- The third line contains N space-separated integers A_1 A_2 … A_N
- The fourth line contains Q space-separated integers G_1 G_2 … G_Q, denoting each GOAT query.

Output Format

For each test case, output Q space-separated integers $\text{GOAT}(G_1)$ $\text{GOAT}(G_2)$ … $\text{GOAT}(G_Q)$

Constraints

$1 \leq G_i \leq N$ for all GOAT queries.

Main Test Set

$1 \leq T \leq 100$
$1 \leq N, Q \leq 10^5$
$1 \leq A_i \leq 10^4$ for all i
The sum of N across all test cases in an input file does not exceed 10^5.
The sum of Q across all test cases in an input file does not exceed 10^5.

Sample Test Cases

Sample Input

```
3
5 1
0 1 1 2 2
6 18 15 4 12
1
11 2
0 3 1 8 3 2 8 1 7 4 4
11 28 33 24 25 16 7 12 9 10 12
8 3
14 3
0 1 1 1 1 2 2 2 3 4 6 6 6 8
18 15 6 14 9 28 12 16 24 5 32 2 51 42
6 2 1
```
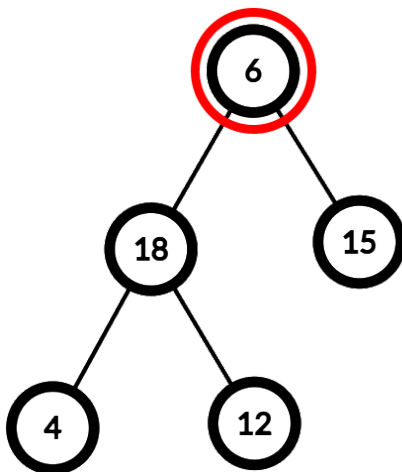
Sample Output

```
36
21 92
137 234 302
```

Main Sample Explanations

Test Case #1:

The input tree looks like this:

The vertex with GOAT a query performed on it is circled in red.

The vertex pairs (u, v) in vertex 1's subtree are listed below:

- (1, 2): $\gcd(A_1, A_2) = \gcd(6, 18) = 6$
- (1, 3): $\gcd(A_1, A_3) = \gcd(6, 15) = 3$
- (1, 4): $\gcd(A_1, A_4) = \gcd(6, 4) = 2$
- (1, 5): $\gcd(A_1, A_5) = \gcd(6, 12) = 6$
- (2, 3): $\gcd(A_2, A_3) = \gcd(18, 15) = 3$
- (2, 4): $\gcd(A_2, A_4) = \gcd(18, 4) = 2$
- (2, 5): $\gcd(A_2, A_5) = \gcd(18, 12) = 6$
- (3, 4): $\gcd(A_3, A_4) = \gcd(15, 4) = 1$
- (3, 5): $\gcd(A_3, A_5) = \gcd(15, 12) = 3$
- (4, 5): $\gcd(A_4, A_5) = \gcd(4, 12) = 4$

GOAT(1) is the sum of these values modulus 998244353, which equals 36.