

Problem 4: CALICO's Corporate Conundrum

15+9+6=30 Points

Problem ID: `managers`

Rank: 2+3+3

Introduction

The year is 2069 and CALICO (which now stands for CALICO International Conglomerate) has expanded their operations! Compared to the two employees it had in 2022, CALICO now benevolently employs hundreds of thousands of low-wage contractors. With such a large company, disputes between the "people" working there often arise, and their managers need to step in to resolve them. Unfortunately, this often leads to managers being overworked!

Problem Statement

Find the largest number of unique disputes a single person is responsible for resolving within the company given the managers of all N of its employees. Each employee is identified using a number counting upwards from 0, with the last employee being assigned the number $N - 1$. The i th employee's manager M_i denotes the employee number of their direct superior. The CEO is assigned employee number 0 and is the only employee whose direct superior is themselves.

A dispute between two employees is resolved by the lowest-level manager that has authority over both of them, whether directly or indirectly. Any employee also has authority over themselves, meaning it's possible for a dispute to be resolved by someone involved in it. Two disputes are unique if they involve at least one different employee.

Input Format

The first line of the input contains a positive integer T denoting the number of test cases that follow. For each test case:

- The first line contains the single positive integer N denoting the number of employees present in the company.
- The second line contains the space-separated sequence of N non-negative integers M_0, M_1, \dots, M_{N-1} denoting the employee number of each individual's manager.
 - M_0 will always be zero to represent the CEO being their own direct superior.

Output Format

For each test case, output a single line containing the largest number of unique disputes a single person is responsible for resolving within the company.

Constraints

$$1 \leq T \leq 100$$

Main Test Set

$$1 \leq N \leq 50$$

The sum of N across all test cases does not exceed 250.

An employee directly manages *at most* two other employees.

Bonus Test Set 1

$$1 \leq N \leq 10^5$$

The sum of N across all test cases does not exceed 10^5 .

An employee directly manages *at most* two other employees.

An employee has at most 10^3 direct or indirect managers.

Bonus Test Set 2

$$1 \leq N \leq 10^5$$

The sum of N across all test cases does not exceed 10^5 .

An employee can directly manage *any* number of employees.

An employee has at most 10^3 direct or indirect managers.

Sample Test Cases

Main Sample Input

```
3
5
0 0 0 1 1
11
0 4 3 0 8 3 2 8 0 7 4
16
0 0 1 1 2 2 3 4 4 5 5 7 8 9 9 10
6
0 0 1 2 3 4
```

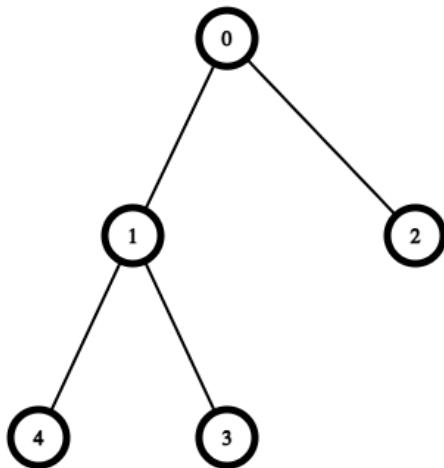
Main Sample Output

```
7
34
41
5
```

Main Sample Explanations

For Test Case #1:

The company structure looks like this:



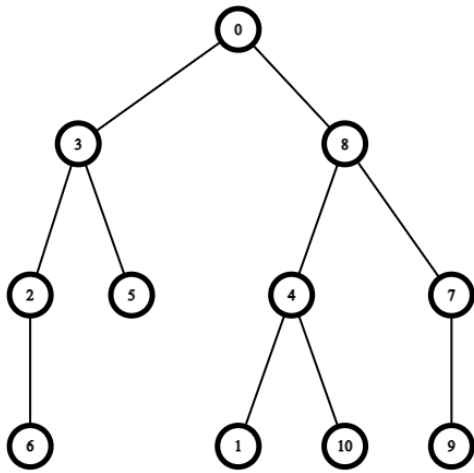
The list of all possible disputes are between:

- 0 and 1: resolved by 0
- 0 and 2: resolved by 0
- 0 and 3: resolved by 0
- 0 and 4: resolved by 0
- 1 and 2: resolved by 0
- 1 and 3: resolved by 1
- 1 and 4: resolved by 1
- 2 and 3: resolved by 0
- 2 and 4: resolved by 0
- 3 and 4: resolved by 1

The maximum number of disputes a single employee is responsible for is 7 (employee 0).

For Test Case #2:

The company structure looks like this:



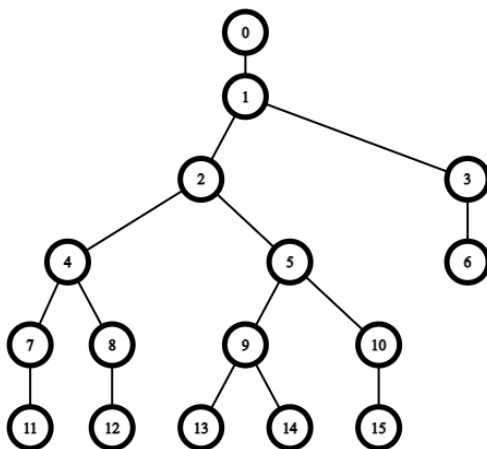
The number of unique disputes each employee is responsible for is as follows:

- Employee 0: 34 disputes
- Employee 1: 0 disputes
- Employee 2: 1 dispute
- Employee 3: 5 disputes
- Employee 4: 3 disputes
- Employee 5: 0 disputes
- Employee 6: 0 disputes
- Employee 7: 1 dispute
- Employee 8: 11 disputes
- Employee 9: 0 disputes
- Employee 10: 0 disputes

The maximum number of disputes a single employee is responsible for is 34 (employee 0).

For Test Case #3:

The company structure looks like this:



The top five employees responsible for the most disputes are:

1. Employee 2 (41 disputes)
2. Employee 1 (38 disputes)
3. Employee 0 (15 disputes)
4. Employee 5 (11 disputes)
5. Employee 4 (8 disputes)

For Test Case #4:

The company structure looks like this:



The top five employees responsible for the most disputes are:

1. Employee 0 (5 disputes)
2. Employee 1 (4 disputes)
3. Employee 2 (3 disputes)
4. Employee 3 (2 disputes)
5. Employee 4 (1 dispute)

Bonus Set 2 Sample Input

```
2
14
0 0 0 0 0 1 1 1 2 3 5 5 5 7
29
0 4 3 0 8 3 2 8 0 7 4 3 3 3 8 7 14 14 8 9 14 10 18 22 22 8 25 26 27
28
```

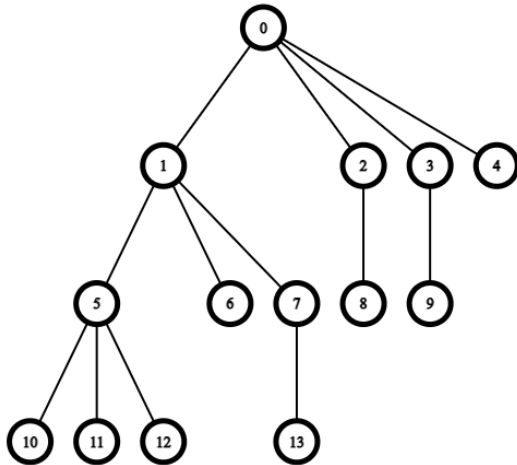
Bonus Set 2 Sample Output

```
61
180
```

Bonus Set 2 Sample Explanations

For Test Case #1:

The company structure looks like this:

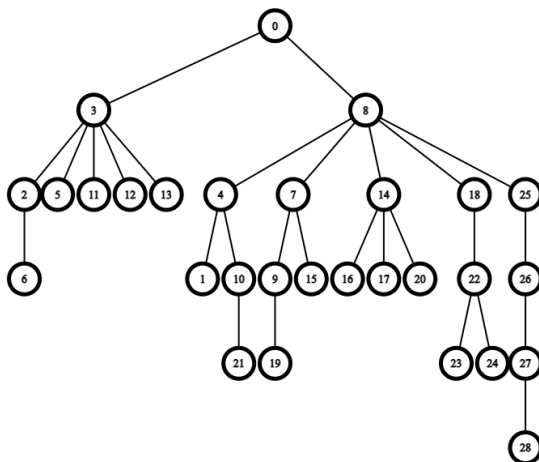


The top three employees responsible for the most disputes are:

1. Employee 0 (61 disputes)
2. Employee 1 (21 disputes)
3. Employee 5 (6 disputes)

For Test Case #2:

The company structure looks like this:



The top four employees responsible for the most disputes are:

1. Employee 8 (180 disputes)
2. Employee 0 (175 disputes)
3. Employee 3 (20 disputes)
4. Employee 14 (6 disputes)